Lesson 1: Interpreting Division of a Fraction by a Whole Number—Visual Models

Classwork

Opening Exercise

A

Write a division sentence to solve each problem.

1. 8 gallons of batter are poured equally into 4 bowls.
   How many gallons of batter are in each bowl?

2. 1 gallon of batter is poured equally into 4 bowls.
   How many gallons of batter are in each bowl?

Write a division sentence *and* draw a model to solve.

3. 3 gallons of batter are poured equally into 4 bowls.
   How many gallons of batter are in each bowl?

B

Write a multiplication sentence to solve each problem.

1. One fourth of an 8-gallon pail is poured out.
   How many gallons are poured out?

2. One fourth of a 1-gallon pail is poured out.
   How many gallons are poured out?

Write a multiplication sentence *and* draw a model to solve.

3. One fourth of a 3-gallon pail is poured out.
   How many gallons are poured out?

Example 1

$\frac{3}{4}$ gallon of batter is poured equally into 2 bowls. How many gallons of batter are in each bowl?

Example 2

$\frac{3}{4}$ pan of lasagna is shared equally by 6 friends. What fraction of the pan will each friend get?

Example 3

A rope of length $\frac{2}{5}$ m is cut into 4 equal cords. What is the length of each cord?

Exercises 1–6

Fill in the blanks to complete the equation. Then, find the quotient and draw a model to support your solution.

1. $\frac{1}{2} \div 3 = \frac{\square}{3} \cdot \frac{1}{2}$

2. $\frac{1}{3} \div 4 = \frac{1}{4} \cdot \frac{1}{\square}$

Find the value of each of the following.

3. $\frac{1}{4} \div 5$

4. $\frac{3}{5} \div 5$

5. $\frac{1}{5} \div 4$

Solve. Draw a model to support your solution.

6. $\frac{3}{5}$ pint of juice is poured equally into 6 glasses. How much juice is in each glass?

Lesson 2: Interpreting Division of a Whole Number by a Fraction—Visual Models

Classwork

Example 1 **Question #_____**

Write it as a division expression.

Write it as a multiplication expression.

Make a rough draft of a model to represent the problem:

As you travel to each model, be sure to answer the following questions:

Original Question	Corresponding Division Expression	Corresponding Multiplication Expression	Write an Equation Showing the Equivalence of the Two Expressions.
1. How many $\frac{1}{2}$ miles are in 12 miles?			
2. How many quarter hours are in 5 hours?			
3. How many $\frac{1}{3}$ cups are in 9 cups?			
4. How many $\frac{1}{8}$ pizzas are in 4 pizzas?			
5. How many one-fifths are in 7 wholes?			

Example 2

Molly has 9 cups of flour. If this is $\frac{3}{4}$ of the number she needs to make bread, how many cups does she need?

a. Construct the tape diagram by reading it backward. Draw a tape diagram and label the unknown.

b. Next, shade in $\frac{3}{4}$.

c. Label the shaded region to show that 9 is equal to $\frac{3}{4}$ of the total.

d. Analyze the model to determine the quotient.

Exercises 1–5

1. A construction company is setting up signs on 2 miles of road. If the company places a sign every $\frac{1}{4}$ mile, how many signs will it use?

2. George bought 4 submarine sandwiches for a birthday party. If each person will eat $\frac{2}{3}$ of a sandwich, how many people can George feed?

3. Miranda buys 6 pounds of nuts. If she puts $\frac{3}{4}$ pound in each bag, how many bags can she make?

4. Margo freezes 8 cups of strawberries. If this is $\frac{2}{3}$ of the total strawberries that she picked, how many cups of strawberries did Margo pick?

5. Regina is chopping up wood. She has chopped 10 logs so far. If the 10 logs represent $\frac{5}{8}$ of all the logs that need to be chopped, how many logs need to be chopped in all?

Lesson 3: Interpreting and Computing Division of a Fraction by a Fraction—More Models

Classwork

Opening Exercise

Draw a model to represent $12 \div 3$.

Create a question or word problem that matches your model.

Example 1

$\frac{8}{9} \div \frac{2}{9}$

Write the expression in unit form, and then draw a model to solve.

Example 2

$\frac{9}{12} \div \frac{3}{12}$

Write the expression in unit form, and then draw a model to solve.

Example 3

$\frac{7}{9} \div \frac{3}{9}$

Write the expression in unit form, and then draw a model to solve.

Exercises 1–6

Write an expression to represent each problem. Then, draw a model to solve.

1. How many fourths are in 3 fourths?

2. $\frac{4}{5} \div \frac{2}{5}$

3. $\frac{9}{4} \div \frac{3}{4}$

4. $\frac{7}{8} \div \frac{2}{8}$

5. $\frac{13}{10} \div \frac{2}{10}$

6. $\frac{11}{9} \div \frac{3}{9}$

Lesson 4: Interpreting and Computing Division of a Fraction by a Fraction—More Models

Classwork

Opening Exercise

Write at least three equivalent fractions for each fraction below.

a. $\frac{2}{3}$

b. $\frac{10}{12}$

Example 1

Molly has $1\frac{3}{8}$ cups of strawberries. She needs $\frac{3}{8}$ cup of strawberries to make one batch of muffins. How many batches can Molly make? Use a model to support your answer.

Example 2

Molly's friend, Xavier, also has $\frac{11}{8}$ cups of strawberries. He needs $\frac{3}{4}$ cup of strawberries to make a batch of tarts. How many batches can he make? Draw a model to support your solution.

Example 3

Find the quotient: $\frac{6}{8} \div \frac{2}{8}$. Use a model to show your answer.

Example 4

Find the quotient: $\frac{3}{4} \div \frac{2}{3}$. Use a model to show your answer.

Exercises 1–5

Find each quotient. Use a model to help you.

1. $\frac{6}{2} \div \frac{3}{4}$

2. $\frac{2}{3} \div \frac{2}{5}$

3. $\frac{7}{8} \div \frac{1}{2}$

4. $\frac{3}{5} \div \frac{1}{4}$

5. $\frac{5}{4} \div \frac{1}{3}$

Lesson 5: Creating Division Stories

Classwork

Opening Exercise

Tape Diagram:

$\frac{8}{9} \div \frac{2}{9}$

Number Line:

Molly's friend, Xavier, also has $\frac{11}{8}$ cups of strawberries. He needs $\frac{3}{4}$ cup of strawberries to make a batch of tarts. How many batches can he make? Draw a model to support your solution.

Example 1

$\frac{1}{2} \div \frac{1}{8}$

Step 1: Decide on an interpretation.

Step 2: Draw a model.

Step 3: Find the answer.

Step 4: Choose a unit.

Step 5: Set up a situation based upon the model.

Exercise 1

Using the same dividend and divisor, work with a partner to create your own story problem. You may use the same unit, but your situation must be unique. You could try another unit such as ounces, yards, or miles if you prefer.

Example 2

$\frac{3}{4} \div \frac{1}{2}$

Step 1: Decide on an interpretation.

Step 2: Draw a diagram.

Step 3: Find the answer.

Step 4: Choose a unit.

Step 5: Set up a situation based on the model.

Exercise 2

Using the same dividend and divisor, work with a partner to create your own story problem. You may use the same unit, but your situation must be unique. You could try another unit such as cups, yards, or miles if you prefer.

Lesson 6: More Division Stories

Classwork

Example 1

Divide $50 \div \frac{2}{3}$.

Step 1: Decide on an interpretation.

Step 2: Draw a model.

Step 3: Find the answer.

Step 4: Choose a unit.

Step 5: Set up a situation based upon the model.

Exercise 1

Using the same dividend and divisor, work with a partner to create your own story problem. You may use the same unit, dollars, but your situation must be unique. You could try another unit, such as miles, if you prefer.

Example 2

Divide $\frac{1}{2} \div \frac{3}{4}$.

Step 1: Decide on an interpretation.

Step 2: Draw a model.

Step 3: Find the answer.

Step 4: Choose a unit.

Step 5: Set up a situation based upon the model.

Exercise 2

Using the same dividend and divisor, work with a partner to create your own story problem. You could try a different unit, such as feet (remember: 12 inches = 1 foot).

Lesson 7: The Relationship Between Visual Fraction Models and Equations

Classwork

Example 1

Model the following using a partitive interpretation.

$\frac{3}{4} \div \frac{2}{5}$

Shade 2 of the 5 sections $\left(\frac{2}{5}\right)$.

Label the part that is known $\left(\frac{3}{4}\right)$.

Make notes below on the math sentences needed to solve the problem.

Example 2

Model the following using a measurement interpretation.

$\frac{3}{5} \div \frac{1}{4}$

Example 3

$\frac{2}{3} \div \frac{3}{4}$

Show the number sentences below.

Lesson 8: Dividing Fractions and Mixed Numbers

Classwork

Example 1: Introduction to Calculating the Quotient of a Mixed Number and a Fraction

a. Carli has $4\frac{1}{2}$ walls left to paint in order for all the bedrooms in her house to have the same color paint. However, she has used almost all of her paint and only has $\frac{5}{6}$ of a gallon left.

How much paint can she use on each wall in order to have enough to paint the remaining walls?

b. Calculate the quotient.

$\frac{2}{5} \div 3\frac{4}{7}$

Exercise: Show your work for the memory game in the boxes provided below.

A.
B.
C.
D.
E.
F.
G.
H.
I.
J.
K.
L.

Lesson 9: Sums and Differences of Decimals

Classwork

Example 1

$25\frac{3}{10} + 376\frac{77}{100}$

Example 2

$426\frac{1}{5} - 275\frac{1}{2}$

Exercises: Calculate each sum or difference.

1. Samantha and her friends are going on a road trip that is $245\frac{7}{50}$ miles long. They have already driven $128\frac{53}{100}$. How much farther do they have to drive?

2. Ben needs to replace two sides of his fence. One side is $367\frac{9}{100}$ meters long, and the other is $329\frac{3}{10}$ meters long. How much fence does Ben need to buy?

3. Mike wants to paint his new office with two different colors. If he needs $4\frac{4}{5}$ gallons of red paint and $3\frac{1}{10}$ gallons of brown paint, how much paint does he need in total?

4. After Arianna completed some work, she figured she still had $78\frac{21}{100}$ pictures to paint. If she completed another $34\frac{23}{25}$ pictures, how many pictures does Arianna still have to paint?

Lesson 10: The Distributive Property and the Products of Decimals

Classwork

Opening Exercise

Calculate the product.

a. $200 \cdot 32.6$

b. $500 \cdot 22.12$

Example 1: Introduction to Partial Products

Use partial products and the distributive property to calculate the product.

$200 \cdot 32.6$

Example 2: Introduction to Partial Products

Use partial products and the distributive property to calculate the area of the rectangular patio shown below.

Exercises

Use the boxes below to show your work for each station.

Station One:
Station Two:
Station Three:
Station Four:
Station Five:

Lesson 11: Fraction Multiplication and the Products of Decimals

Classwork

Exploratory Challenge

Solve each problem. Your groups also need to prove that the decimal in the product is located in the correct place.

a. Calculate the product. $34.62 \cdot 12.8$

b. Xavier earns $\$11.50$ per hour working at the grocery store. Last week, Xavier worked for 13.5 hours. How much money did Xavier earn last week? Remember to round to the nearest penny.

Discussion

Record notes from the Discussion in the box above.

Exercises

1. Calculate the product. $324.56 \cdot 54.82$

2. Kevin spends $\$11.25$ on lunch every week during the school year. If there are 35.5 weeks during the school year, how much does he spend on lunch for the school year? Round to the nearest penny.

3. Gunnar's car gets 22.4 miles per gallon, and his gas tank can hold 17.82 gallons of gas. How many miles can Gunnar travel if he uses all of the gas in the gas tank?

4. The principal of East High School wants to buy a new cover for the sand pit used in the long-jump competition. He measured the sand pit and found that the length is 29.2 feet and the width is 9.8 feet. What will the area of the new cover be?

Lesson 12: Estimating Digits in a Quotient

Classwork

Discussion

Divide 150 by 30.

Example: Rounding to a One-Digit Arithmetic Fact

$26{,}897 \div 38$

Exercises 1–4: Round to estimate the quotient. Then, compute the quotient using a calculator, and compare the estimation to the quotient.

1. $2{,}970 \div 11$

   a. Round to a one-digit arithmetic fact. Estimate the quotient.

   b. Use a calculator to find the quotient. Compare the quotient to the estimate.

2. $4{,}752 \div 12$

   a. Round to a one-digit arithmetic fact. Estimate the quotient.

   b. Use a calculator to find the quotient. Compare the quotient to the estimate.

3. $11{,}647 \div 19$

   a. Round to a one-digit arithmetic fact. Estimate the quotient.

   b. Use a calculator to find the quotient. Compare the quotient to the estimate.

4. $40{,}644 \div 18$

   a. Round to a one-digit arithmetic fact. Estimate the quotient.

   b. Use a calculator to find the quotient. Compare the quotient to the estimate.

Lesson 13: Dividing Multi-Digit Numbers Using the Algorithm

Classwork

Example 1

Divide $70{,}072 \div 19$.

a. Estimate:

b. Create a table to show the multiples of 19.

Multiples of 19

c. Use the algorithm to divide $70{,}072 \div 19$. Check your work.

$$19 \overline{)\,70072}$$

Example 2

Divide $14{,}175 \div 315$.

a. Estimate:

b. Use the algorithm to divide $14{,}175 \div 315$. Make a list of multiples of 315 first. Check your work.

Exercises 1–4

1. $48{,}312 \div 18$
   a. Estimate:

   b. Divide using the algorithm. Check your answer.

2. $281{,}886 \div 33$
   a. Estimate:

   b. Divide using the algorithm. Check your answer.

3. $2{,}295{,}517 \div 37$

   a. Estimate:

   b. Divide using the algorithm. Check your answer.

4. $952{,}448 \div 112$

   a. Estimate:

   b. Divide using the algorithm. Check your answer.

Lesson 14: The Division Algorithm—Converting Decimal Division into Whole Number Division Using Fractions

Classwork

Opening Exercise

Divide $\frac{1}{2} \div \frac{1}{10}$. Use a tape diagram to support your reasoning.

Relate the model to the invert and multiply rule.

Example 1

Evaluate the expression. Use a tape diagram to support your answer.

$0.5 \div 0.1$

Rewrite $0.5 \div 0.1$ as a fraction.

Create an equivalent fraction where the divisor is a whole number.

Exercises 1–3

Convert the decimal division expressions to fractional division expressions in order to create whole number divisors. You do not need to find the quotients. Explain the place value movement of the digits. **The first exercise has been completed for you.**

1. $18.6 \div 2.3$

   $\frac{18.6}{2.3} \times \frac{10}{10} = \frac{186}{23}$

   $186 \div 23$

   I created an equivalent fraction by multiplying by $\frac{10}{10}$. Each digit in the numerator and denominator moved one place value to the left because they both grew 10 times larger.

2. $14.04 \div 4.68$

3. $0.162 \div 0.036$

Example 2

Evaluate the expression. First, convert the decimal division expression to a fractional division expression in order to create an equivalent fraction where the divisor is a whole number.

$25.2 \div 0.72$

Use the division algorithm to find the quotient.

Exercises 4–7

Convert the decimal division expressions to fractional division expressions in order to create equivalent fractions where the divisors are whole numbers. Compute the quotients using the division algorithm. Check your work with a calculator.

4. $2{,}000 \div 3.2$

5. $3{,}581.9 \div 4.9$

6. $893.76 \div 0.21$

7. $6.194 \div 0.326$

Example 3

A plane travels $3{,}625.26$ miles in 6.9 hours. What is the plane's unit rate?

Represent this situation with a fraction. Estimate the quotient.

Create an equivalent fraction where the divisor is a whole number.

Use the division algorithm to find the quotient.

Use multiplication to check your work.

Lesson 16: Even and Odd Numbers

Classwork

Opening Exercise

a. What is an even number?

b. List some examples of even numbers.

c. What is an odd number?

d. List some examples of odd numbers.

What happens when we add two even numbers? Do we always get an even number?

Exercises 1–3

1. Why is the sum of two even numbers even?
   a. Think of the problem $12 + 14$. Draw dots to represent each number.

   b. Circle pairs of dots to determine if any of the dots are left over.

   c. Is this true every time two even numbers are added together? Why or why not?

2. Why is the sum of two odd numbers even?
   a. Think of the problem $11 + 15$. Draw dots to represent each number.

   b. Circle pairs of dots to determine if any of the dots are left over.

   c. Is this true every time two odd numbers are added together? Why or why not?

3. Why is the sum of an even number and an odd number odd?

   a. Think of the problem $14 + 11$. Draw dots to represent each number.

   b. Circle pairs of dots to determine if any of the dots are left over.

   c. Is this true every time an even number and an odd number are added together? Why or why not?

   d. What if the first addend is odd and the second is even? Is the sum still odd? Why or why not? For example, if we had $11 + 14$, would the sum be odd?

Adding:

-
-
-

Exploratory Challenge/Exercises 4–6

4. The product of two even numbers is even.

5. The product of two odd numbers is odd.

6. The product of an even number and an odd number is even.

Multiplying:

-
-
-

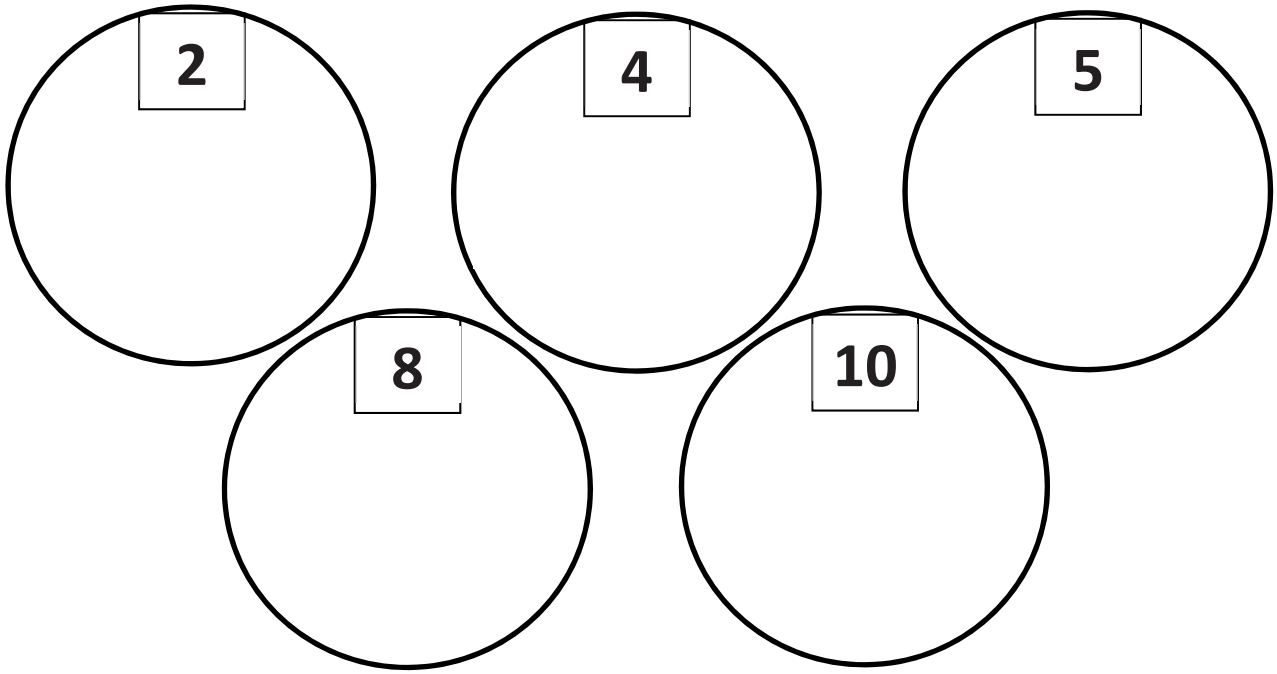
Lesson 17: Divisibility Tests for 3 and 9

Classwork

Opening Exercise

Below is a list of 10 numbers. Place each number in the circle(s) that is a factor of the number. Some numbers can be placed in more than one circle. For example, if 32 were on the list, it would be placed in the circles with 2, 4, and 8 because they are all factors of 32.

24; 36; 80; 115; 214; 360; 975; 4,678; 29,785; 414,940



Discussion

- Divisibility rule for 2:

- Divisibility rule for 4:

- Divisibility rule for 5:

- Divisibility rule for 8:

- Divisibility rule for 10:

- Decimal numbers with fraction parts do not follow the divisibility tests.

- Divisibility rule for 3:

- Divisibility rule for 9:

Example 1

This example shows how to apply the two new divisibility rules we just discussed.

Explain why 378 is divisible by 3 and 9.

a. Expand 378.

b. Decompose the expression to factor by 9.

c. Factor the 9.

d. What is the sum of the three digits?

e. Is 18 divisble by 9?

f. Is the number 378 divisible by 9? Is the number 378 divisible by 3? Why or why not?

Example 2

Is 3,822 divisible by 3 or 9? Why or why not?

Exercises 1–4

Circle ALL the numbers that are factors of the given number. Complete any necessary work in the space provided.

1. 2,838 is divisible by

   3

   9

   4

   Explain your reasoning for your choice(s).

2. 34,515 is divisible by

   3

   9

   5

   Explain your reasoning for your choice(s).

3. 4,320 is divisible by

   3

   9

   10

   Explain your reasoning for your choice(s).

4. 6,240 is divisible by

   3

   9

   8

   Explain your reasoning for your choice(s).

Lesson 18: Least Common Multiple and Greatest Common Factor

Classwork

Example 1: Greatest Common Factor

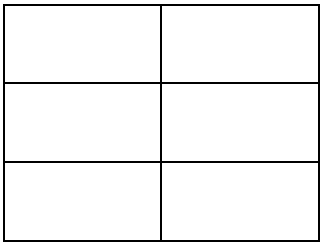
1. Find the greatest common factor of 12 and 18.

   - Listing these factor pairs in order helps ensure that no common factors are missed. Start with 1 multiplied by the number.
   - Circle all factors that appear on both lists.
   - Place a triangle around the greatest of these common factors.

GCF $(12, 18)$

12

18

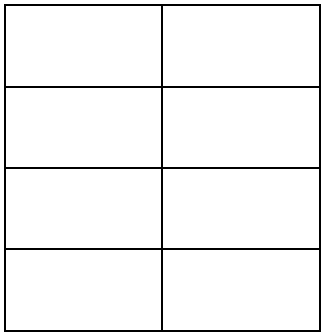


2. Find the greatest common factor of 24 and 36.

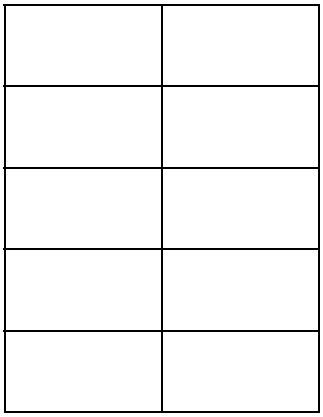
   - List all the factor pairs.
   - Circle all factors that appear on both lists.
   - Place a triangle around the greatest of these common factors.

GCF $(24, 36)$

24



36



Exercises:

1. GCF $(30, 50)$
2. GCF $(45, 60)$
3. GCF $(96, 144)$

4. There are 18 girls and 24 boys who want to participate in a Trivia Challenge. If each team must have the same ratio of girls and boys, what is the greatest number of teams that can enter? Find how many boys and girls each team would have.

5. Is the GCF of a pair of numbers ever equal to one of the numbers? Explain.

6. Is the GCF of a pair of numbers ever greater than both numbers? Explain.

Example 2: Least Common Multiple

1. Find the least common multiple of 12 and 18.

   LCM $(12, 18)$

   Write the first 10 multiples of 12.

   Write the first 10 multiples of 18.

   Circle the multiples that appear on both lists.

   Put a rectangle around the least of these common multiples.

2. Find the least common multiple of 6 and 14.

   LCM $(6, 14)$

   Write the first 10 multiples of 6.

   Write the first 10 multiples of 14.

   Circle the multiples that appear on both lists.

   Put a rectangle around the least of these common multiples.

Exercises:

1. LCM $(9, 12)$

2. LCM $(4, 30)$

3. LCM $(20, 50)$

4. Hot dogs come packed 10 in a package. Hot dog buns come packed 8 in a package. If we want one hot dog for each bun for a picnic with none left over, what is the least amount of each we need to buy? How many packages of each item would we have to buy?

5. Is the LCM of a pair of numbers ever equal to one of the numbers? Explain.

6. Is the LCM of a pair of numbers ever less than both numbers? Explain.

Example 3: Prime Factorization

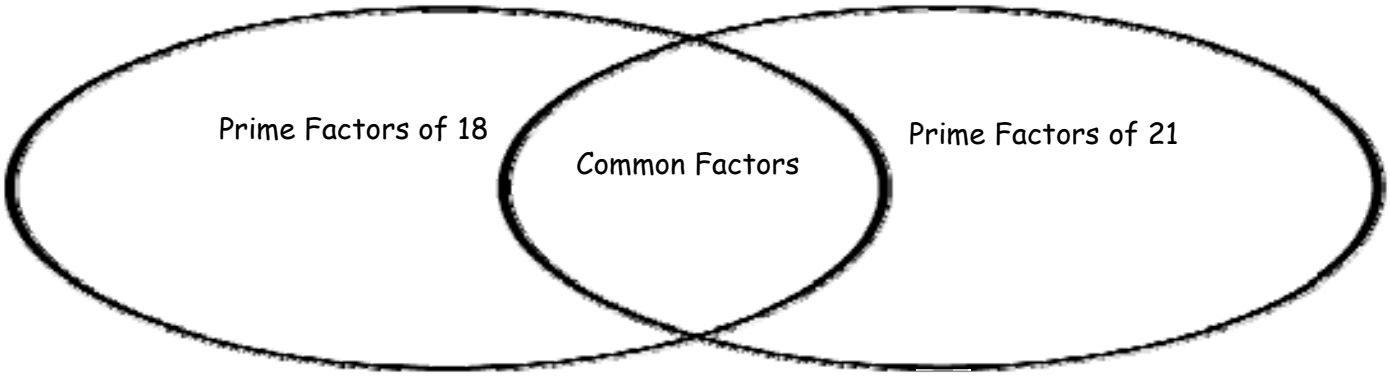
Find the greatest common factor of 18 and 21 using prime factorization.

GCG $(18, 21)$

Write the prime factorization of 18.

Write the prime factorization of 21.

Write the prime factors for each number into the appropriate sections of the Venn Diagram.



Multiply the common factors to get the GCF.

Exercises:

Find the great common factor for each pair using prime factorization.

1. GCF $(30, 50)$

2. GCF $(45, 60)$

3. GCF $(42, 70)$

4. GCF $(96, 144)$

5. Would you rather find all the factors of a number or find all the prime factors of a number? Why?

Example 4: Distributive Property

1. Find the greatest common factor of 18 and 21.

   GCG $(18, 21)$

   Rewrite the sum using the distributive property and the GCF.

   $18 + 21 =$

2. Find the greatest common factor of 36 and 24.

   GCG $(36, 24)$

   Rewrite the sum using the distributive property and the GCF.

   $36 - 24 =$

Exercises:

Find the GCF from the two numbers, and rewrite the sum or difference using the distributive property.

1. $12 + 18 =$

2. $36 - 27 =$

3. $16 + 72 =$

4. $44 - 33 =$

5. Choose any numbers for n, a, and b to create an example for each equation below.

$n(a) + n(b) = n(a + b)$

$n(a) - n(b) = n(a - b)$