

# Lesson 1: Ratios

## Classwork

### Example 1

The coed soccer team has four times as many boys on it as it has girls. We say the ratio of the number of boys to the number of girls on the team is 4:1. We read this as *four to one*. Create a table to show how many boys and how many girls could be on the team.

Suppose the ratio of the number of boys to the number of girls on the team is 3:2. Create a table to show how many boys and how many girls could be on a team with this ratio.

Make a tape diagram to show the ratio of the number of boys to the number of girls on the team is 3:2

**Example 2: Class Ratios**

Write the ratio of the number of boys to the number of girls in our class.

Write the ratio of the number of girls to the number of boys in our class.

Record a ratio for each of the examples the teacher provides.

1. \_\_\_\_\_

2. \_\_\_\_\_

3. \_\_\_\_\_

4. \_\_\_\_\_

5. \_\_\_\_\_

6. \_\_\_\_\_

**Exercise 1**

My own ratio compares \_\_\_\_\_ to

\_\_\_\_\_.

My ratio is \_\_\_\_\_.

**Exercise 2**

Using words, describe a ratio that represents each ratio below.

a. 1 to 12 \_\_\_\_\_  
\_\_\_\_\_.

b. 12:1 \_\_\_\_\_  
\_\_\_\_\_.

c. 2 to 5 \_\_\_\_\_  
\_\_\_\_\_.

d. 5 to 2 \_\_\_\_\_  
\_\_\_\_\_.

e. 10:2 \_\_\_\_\_  
\_\_\_\_\_.

f. 2:10 \_\_\_\_\_  
\_\_\_\_\_.

# Lesson 2: Ratios

## Classwork

### Exercise 1

Come up with two examples of ratio relationships that are interesting to you.

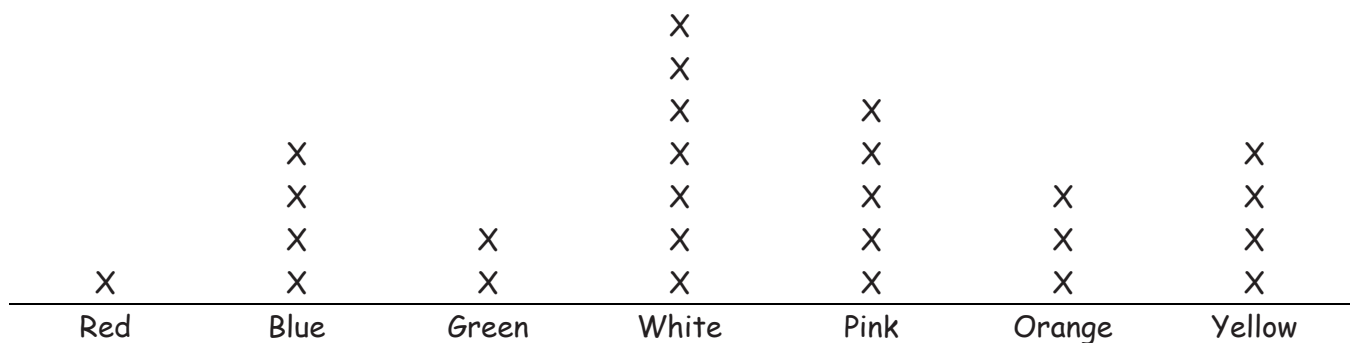
1.

2.

### Exploratory Challenge

A T-shirt making company surveyed teenage girls on their favorite T-shirt color to guide the company's decisions about how many of each color T-shirt they should make. The results of the survey are shown here.

**Favorite T-Shirt Colors of Teenage Girls Surveyed**



### Exercises for Exploratory Challenge

1. Describe a ratio relationship, in the context of this survey, for which the ratio is 3: 5.

2. For each ratio relationship given, fill in the ratio it is describing.

Description of the Ratio Relationship (Underline or highlight the words or phrases that indicate the description is a ratio.)	Ratio
For every 7 white T-shirts they make, they should make 4 yellow T-shirts. The ratio of the number of white T-shirts to the number of yellow T-shirts should be ...	
For every 4 yellow T-shirts they make, they should make 7 white T-shirts. The ratio of the number of yellow T-shirts to the number of white T-shirts should be ...	
The ratio of the number of girls who liked an orange T-shirt best to the number of girls who liked a blue, green, or pink T-shirt was...	
For each red T-shirt they make, they should make 4 blue T-shirts. The ratio of the number of red T-shirts to the number of blue T-shirts should be ...	
They should purchase 4 bolts of yellow fabric for every 3 bolts of orange fabric. The ratio of the number of bolts of yellow fabric to the number of bolts of orange fabric should be ...	
The ratio of the number of girls who chose blue or green as their favorite to the number of girls who chose pink or red as their favorite was ...	
Three out of every 26 T-shirts they make should be orange. The ratio of the number of orange T-shirts to the total number of T-shirts should be ...	

3. For each ratio given, fill in a description of the ratio relationship it could describe, using the context of the survey.

Description of the Ratio Relationship (Underline or highlight the words or phrases that indicate your example is a ratio.)	Ratio
	4 to 3
	3:4
	19:7
	7 to 26

## Lesson 3: Equivalent Ratios

### Classwork

#### Exercise 1

Write a one-sentence story problem about a ratio.

Write the ratio in two different forms.

#### Exercise 2

Shanni and Mel are using ribbon to decorate a project in their art class. The ratio of the length of Shanni's ribbon to the length of Mel's ribbon is 7:3.

a. Represent this ratio in a table.

b. Draw a tape diagram to represent this ratio. What ratios can we say are equivalent to 7:3?

**Exercise 3**

Mason and Laney ran laps to train for the long-distance running team. The ratio of the number of laps Mason ran to the number of laps Laney ran was 2 to 3.

- If Mason ran 4 miles, how far did Laney run? Draw a tape diagram to demonstrate how you found the answer.
- If Laney ran 930 meters, how far did Mason run? Draw a tape diagram to determine how you found the answer.
- What ratios can we say are equivalent to 2:3?

**Exercise 4**

Josie took a long multiple-choice, end-of-year vocabulary test. The ratio of the number of problems Josie got incorrect to the number of problems she got correct is 2:9.

- If Josie missed 8 questions, how many did she get correct? Draw a tape diagram to demonstrate how you found the answer.

- b. If Josie missed 20 questions, how many did she get correct? Draw a tape diagram to demonstrate how you found the answer.
- c. What ratios can we say are equivalent to 2:9?
- d. Come up with another possible ratio of the number Josie got incorrect to the number she got correct.
- e. How did you find the numbers?
- f. Describe how to create equivalent ratios.



## Lesson 4: Equivalent Ratios

### Classwork

#### Example 1

The morning announcements said that two out of every seven sixth-grade students in the school have an overdue library book. Jasmine said, "That would mean 24 of us have overdue books!" Grace argued, "No way. That is way too high." How can you determine who is right?

#### Exercise 1

Decide whether or not each of the following pairs of ratios is equivalent.

- If the ratios are not equivalent, find a ratio that is equivalent to the first ratio.
- If the ratios are equivalent, identify the nonzero number,  $c$ , that could be used to multiply each number of the first ratio by in order to get the numbers for the second ratio.

a. 6:11 and 42:88

\_\_\_\_\_ Yes, the value,  $c$ , is \_\_\_\_\_.

\_\_\_\_\_ No, an equivalent ratio would be \_\_\_\_\_.

b. 0:5 and 0:20

\_\_\_\_\_ Yes, the value,  $c$ , is \_\_\_\_\_.

\_\_\_\_\_ No, an equivalent ratio would be \_\_\_\_\_.

**Exercise 2**

In a bag of mixed walnuts and cashews, the ratio of the number of walnuts to the number of cashews is 5:6. Determine the number of walnuts that are in the bag if there are 54 cashews. Use a tape diagram to support your work. Justify your answer by showing that the new ratio you created of the number of walnuts to the number of cashews is equivalent to 5:6.

## Lesson 5: Solving Problems by Finding Equivalent Ratios

### Classwork

#### Example 1

A County Superintendent is interested in the numbers of different types of vehicles that people own within his county. In the month of August, a total of 192 registrations were purchased for passenger cars and pickup trucks at the Department of Motor Vehicles (DMV). The DMV reported that in the month of August, for every 5 passenger cars registered, there were 7 pickup trucks registered. How many of each type of vehicle were registered in the county in the month of August?

- Using the information in the problem, write four different ratios and describe the meaning of each.
- Make a tape diagram that represents the quantities in the part-to-part ratios that you wrote.
- How many equal-sized parts does the tape diagram consist of?
- What total quantity does the tape diagram represent?

e. What value does each individual part of the tape diagram represent?

f. How many of each type of vehicle were registered in August?

### Example 2

The Superintendent is also interested in the numbers of commercial vehicles that frequently use the county's highways. He obtains information from the DMV for the month of September and finds that for every 14 non-commercial vehicles, there were 5 commercial vehicles. If there were 108 more non-commercial vehicles than commercial vehicles, how many of each type of vehicle frequently use the county's highways during the month of September?

**Exercises**

1. The ratio of the number of people who own a smartphone to the number of people who own a flip phone is 4:3. If 500 more people own a smartphone than a flip phone, how many people own each type of phone?
2. Sammy and David were selling water bottles to raise money for new football uniforms. Sammy sold 5 water bottles for every 3 water bottles David sold. Together they sold 160 water bottles. How many did each boy sell?
3. Ms. Johnson and Ms. Siple were folding report cards to send home to parents. The ratio of the number of report cards Ms. Johnson folded to the number of report cards Ms. Siple folded is 2:3. At the end of the day, they folded a total of 300 report cards. How many did each person fold?
4. At a country concert, the ratio of the number of boys to the number of girls is 2:7. If there are 250 more girls than boys, how many boys are at the concert?

## Lesson 6: Solving Problems by Finding Equivalent Ratios

### Classwork

#### Exercises

1. The Business Direct Hotel caters to people who travel for different types of business trips. On Saturday night there is not a lot of business travel, so the ratio of the number of occupied rooms to the number of unoccupied rooms is 2:5. However, on Sunday night the ratio of the number of occupied rooms to the number of unoccupied rooms is 6:1. If the hotel has 432 occupied rooms on Sunday night, how many unoccupied rooms does it have on Saturday night?
  
2. Peter is working out by completing sit-ups and push-ups. Originally, he was completing five sit-ups for every three push-ups, but then he injured his shoulder. After the injury, he completed the same number of repetitions as he did before his injury, but he completed seven sit-ups for every one push-up. During a training session after his injury, Peter completed eight push-ups. How many push-ups was Peter completing before his injury?
  
3. Tom and Rob are brothers who like to make bets about the outcomes of different contests between them. Before the last bet, the ratio of the amount of Tom's money to the amount of Rob's money was 4:7. Rob lost the latest competition, and now the ratio of the amount of Tom's money to the amount of Rob's money is 8:3. If Rob had \$280 before the last competition, how much does Rob have now that he lost the bet?

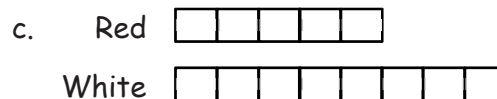
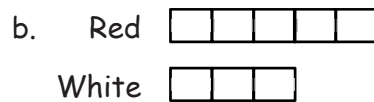
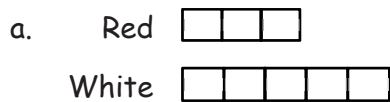
4. A sporting goods store ordered new bikes and scooters. For every 3 bikes ordered, 4 scooters were ordered. However, bikes were way more popular than scooters, so the store changed its next order. The new ratio of the number of bikes ordered to the number of scooters ordered was 5:2. If the same amount of sporting equipment was ordered in both orders and 64 scooters were ordered originally, how many bikes were ordered as part of the new order?
5. During first semester, the ratio of the number of students in art class to the number of students in gym class was 2:7. However, the art classes were really small, and the gym classes were large, so the principal changed students' classes for second semester. In second semester, the ratio of the number of students in art class to the number of students in gym class was 5:4. If 75 students were in art class second semester, how many were in art class and gym class first semester?
6. Jeanette wants to save money, but she has not been good at it in the past. The ratio of the amount of money in Jeanette's savings account to the amount of money in her checking account was 1:6. Because Jeanette is trying to get better at saving money, she moves some money out of her checking account and into her savings account. Now, the ratio of the amount of money in her savings account to the amount of money in her checking account is 4:3. If Jeanette had \$936 in her checking account before moving money, how much money does Jeanette have in each account after moving money?

# Lesson 7: Associated Ratios and the Value of a Ratio

## Classwork

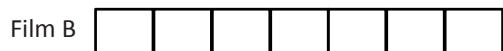
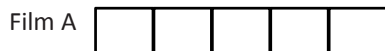
### Example 1

Which of the following correctly models that the number of red gumballs is  $\frac{5}{3}$  the number of white gumballs?



### Example 2

The duration of two films are modeled below.



a. The ratio of the length of Film A to the length of Film B is \_\_\_\_\_ : \_\_\_\_\_.

b. The length of Film A is  $\frac{\square}{\square}$  of the length of Film B.

c. The length of Film B is  $\frac{\square}{\square}$  of the length of Film A.





**Exercise 2**

A food company that produces creamy peanut butter decides to try out a peanut butter that is crunchy. The company hosts a sampling of its new product at grocery stores and finds that 5 out of every 9 customers prefer the new crunchy version to the creamy peanut butter.

- a. Let's make a list of ratios that might be relevant for this situation.
  - i. The ratio of number preferring crunchy to total number surveyed is \_\_\_\_\_.
  - ii. The ratio of number preferring creamy to the total number surveyed is \_\_\_\_\_.
  - iii. The ratio of number preferring creamy to number preferring crunchy is \_\_\_\_\_.
  - iv. The ratio of number preferring crunchy to number preferring creamy is \_\_\_\_\_.
  
- b. Use the value of each ratio to make multiplicative comparisons for each of the ratios below.
  - i. The number preferring crunchy is \_\_\_\_\_ of the total number surveyed.
  - ii. The number preferring creamy is \_\_\_\_\_ of the total number surveyed.
  - iii. The number preferring creamy is \_\_\_\_\_ of those preferring crunchy.
  - iv. The number preferring crunchy is \_\_\_\_\_ of those preferring creamy.
  
- c. If the company is planning to produce 90,000 containers of peanut butter, how many of these containers should be crunchy and how many containers should be creamy?

- d. If the company decides to produce 2,000 containers of creamy peanut butter, how many containers of crunchy peanut butter would it produce?
- e. If the company decides to produce 10,000 containers of crunchy peanut butter, how many containers of creamy peanut butter would it produce?
- f. If the company decides to only produce 3,000 containers of crunchy peanut butter, how many containers of creamy peanut butter would it produce?

## Lesson 8: Equivalent Ratios Defined Through the Value of a Ratio

### Classwork

#### Exercise 1

Circle any equivalent ratios from the list below.

Ratio: 1:2

Ratio: 5:10

Ratio: 6:16

Ratio: 12:32

Find the value of the following ratios, leaving your answer as a fraction, but rewrite the fraction using the largest possible unit.

Ratio: 1:2                      Value of the Ratio:

Ratio: 5:10                     Value of the Ratio:

Ratio: 6:16                     Value of the Ratio:

Ratio: 12:32                    Value of the Ratio:

What do you notice about the value of the equivalent ratios?

#### Exercise 2

Juliet says that if two ratios are equivalent, then their values will be the same. Marcus disagrees. Can you help him come up with a counter example to Juliet's statement? If not, explain why Juliet is correct.



# Lesson 9: Tables of Equivalent Ratios

## Classwork

### Example 1

To make paper mache, the art teacher mixes water and flour. For every two cups of water, she needs to mix in three cups of flour to make the paste. Find equivalent ratios for the ratio relationship 2 cups of water to 3 cups of flour. Represent the equivalent ratios in the table below:

Cups of Water	Cups of Flour

### Example 2

Javier has a new job designing websites. He is paid at a rate of \$700 for every 3 pages of web content that he builds. Create a ratio table to show the total amount of money Javier has earned in ratio to the number of pages he has built.

Total Pages Built								
Total Money Earned								

Javier is saving up to purchase a used car that costs \$4,200. How many web pages will Javier need to build before he can pay for the car?

**Exercise 1**

Spraying plants with cornmeal juice is a natural way to prevent fungal growth on the plants. It is made by soaking cornmeal in water, using two cups of cornmeal for every nine gallons of water. Complete the ratio table to answer the questions that follow.

Cups of Cornmeal	Gallons of Water

- How many cups of cornmeal should be added to 45 gallons of water?
- Paul has only 8 cups of cornmeal. How many gallons of water should he add if he wants to make as much cornmeal juice as he can?
- What can you say about the values of the ratios in the table?

**Exercise 2**

James is setting up a fish tank. He is buying a breed of goldfish that typically grows to be 12 inches long. It is recommended that there be 1 gallon of water for every inch of fish length in the tank. What is the recommended ratio of gallons of water per fully-grown goldfish in the tank?

Complete the ratio table to help answer the following questions:

Number of Fish	Gallons of Water

- What size tank (in gallons) is needed for James to have 5 full-grown goldfish?
- How many fully-grown goldfish can go in a 40-gallon tank?
- What can you say about the values of the ratios in the table?



# Lesson 10: The Structure of Ratio Tables—Additive and Multiplicative

## Classwork

### Exploratory Challenge

Imagine that you are making a fruit salad. For every quart of blueberries you add, you would like to put in 3 quarts of strawberries. Fill in the three ratio tables below for the amounts of strawberries you would use based on the given amounts of blueberries.

Table 1	
Quarts of Blueberries	Quarts of Strawberries
1	
2	
3	
4	
5	

Table 2	
Quarts of Blueberries	Quarts of Strawberries
10	
20	
30	
40	
50	

Table 3	
Quarts of Blueberries	Quarts of Strawberries
100	
200	
300	
400	
500	

a. Describe any patterns you see in the tables. Be specific in your descriptions.

b. How are the amounts of blueberries and strawberries related to each other?

- c. How are the values in the Blueberries column related to each other?
- d. How are the values in the Strawberries column related to each other?
- e. If we know we want to add 7 quarts of blueberries to the fruit salad in Table 1, how can we use the table to help us determine how many strawberries to add?
- f. If we know we used 70 quarts of blueberries to make our salad, how can we use a ratio table to find out how many quarts of strawberries were used?

**Exercise 1**

The following tables were made incorrectly. Find the mistakes that were made, create the correct ratio table, and state the ratio that was used to make the correct ratio table.

a.

Hours	Pay in Dollars
3	24
5	40
7	52
9	72

Hours	Pay in Dollars

Ratio \_\_\_\_\_

b.

Blue	Yellow
1	5
4	8
7	11
10	14

Blue	Yellow

Ratio \_\_\_\_\_

# Lesson 11: Comparing Ratios Using Ratio Tables

## Classwork

### Example 1

Create four equivalent ratios (2 by scaling up and 2 by scaling down) using the ratio 30 to 80.

Write a ratio to describe the relationship shown in the table.

<b>Hours</b>	2	5	6	10
<b>Number of Pizzas Sold</b>	16	40	48	80

### Exercise 1

The following tables show how many words each person can text in a given amount of time. Compare the rates of texting for each person using the ratio table.

Michaela

<b>Minutes</b>	3	5	7	9
<b>Words</b>	150	250	350	450

Jenna

<b>Minutes</b>	2	4	6	8
<b>Words</b>	90	180	270	360

Maria

<b>Minutes</b>	3	6	9	12
<b>Words</b>	120	240	360	480

Complete the table so that it shows Max has a texting rate of 55 words per minute.

<b>Minutes</b>				
<b>Words</b>				

**Exercise 2: Making Juice (Comparing Juice to Water)**

- a. The tables below show the comparison of the amount of water to the amount of juice concentrate (JC) in grape juice made by three different people.

Laredo's Juice		
Water	JC	Total
12	4	16
15	5	20
21	7	28
45	15	60

Franca's Juice		
Water	JC	Total
10	2	12
15	3	18
25	5	30
40	8	48

Milton's Juice		
Water	JC	Total
8	2	10
16	4	20
24	6	30
40	10	50

Put the juices in order from the juice containing the most water to the juice containing the least water.

\_\_\_\_\_

Explain how you used the values in the table to determine the order.

What ratio was used to create each table?

Laredo: \_\_\_\_\_

Franca: \_\_\_\_\_

Milton: \_\_\_\_\_

Explain how the ratio could help you compare the juices.

b. The next day, each person made juice again, but this time they were making apple juice.

Laredo's Juice		
Water	JC	Total
12	2	14
18	3	21
30	5	35
42	7	49

Franca's Juice		
Water	JC	Total
15	6	21
20	8	28
35	14	49
50	20	70

Milton's Juice		
Water	JC	Total
16	6	22
24	9	33
40	15	55
64	24	88

Put the juices in order from the strongest apple taste to the weakest apple taste.

\_\_\_\_\_

Explain how you used the values in the table to determine the order.

What ratio was used to create each table?

Laredo: \_\_\_\_\_ Franca: \_\_\_\_\_ Milton: \_\_\_\_\_

Explain how the ratio could help you compare the juices.

How was this problem different than the grape juice questions in part (a)?

- c. Max and Sheila are making orange juice. Max has mixed 15 cups of water with 4 cups of juice concentrate. Sheila has made her juice by mixing 8 cups water with 3 cups of juice concentrate. Compare the ratios of juice concentrate to water using ratio tables. State which beverage has stronger orange taste.
- d. Victor is making recipes for smoothies. His first recipe calls for 2 cups of strawberries and 7 cups of other ingredients. His second recipe says that 3 cups of strawberries are combined with 9 cups of other ingredients. Which smoothie recipe has more strawberries compared to other ingredients? Use ratio tables to justify your answer.

## Lesson 12: From Ratio Tables to Double Number Line Diagrams

### Classwork

#### Exercise 2

The amount of sugary beverages Americans consume is a leading health concern. For a given brand of cola, a 12 oz. serving of cola contains about 40 g of sugar. Complete the ratio table, using the given ratio to find equivalent ratios.

Cola (ounces)		12	
Sugar (grams)		40	

#### Exercise 3

A 1 L bottle of cola contains approximately 34 fluid ounces. How many grams of sugar would be in a 1 L bottle of the cola? Explain and show how to arrive at the solution.





**Exercise 4**

A school cafeteria has a restriction on the amount of sugary drinks available to students. Drinks may not have more than 25 g of sugar. Based on this restriction, what is the largest size cola (in ounces) the cafeteria can offer to students?



**Exercise 5**

Shontelle solves three math problems in four minutes.

a. Use this information to complete the table below.

<b>Number of Questions</b>	3	6	9	12	15	18	21	24	27	30
<b>Number of Minutes</b>										

b. Shontelle has soccer practice on Thursday evening and has a half hour before practice to work on her math homework and talk to her friends. She has 20 math questions for homework, and she wants to complete them before talking with her friends. How many minutes will Shontelle have left after completing her math homework to talk to her friends?

Use a double number line diagram to support your answer, and show all work.

# Lesson 14: From Ratio Tables, Equations, and Double Number Lines to Plots on the Coordinate Plane

## Classwork

Kelli is traveling by train with her soccer team from Yonkers, NY to Morgantown, WV for a tournament. The distance between Yonkers and Morgantown is 400 miles. The trip will take 8 hours. Here is the train schedule:

Leaving Yonkers, New York	
Destination	Distance
Allentown, PA	100 miles
Carlisle, PA	200 miles
Berkeley Springs, WV	300 miles
Morgantown, WV	400 miles

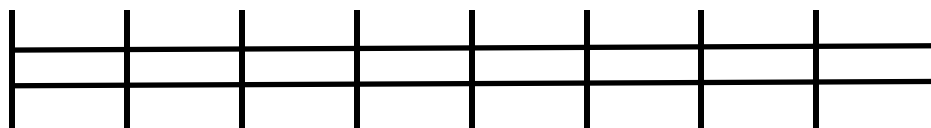
Leaving Morgantown, WV	
Destination	Distance
Berkeley Springs, WV	100 miles
Carlisle, PA	200 miles
Allentown, PA	300 miles
Yonkers, NY	400 miles

## Exercises

1. Create a table to show the total time it will take Kelli's team to travel from Yonkers to each town listed in the schedule and back home again. Assume that the ratio of the amount of time traveled to the distance traveled is the same for each town.

Hours								
Miles								

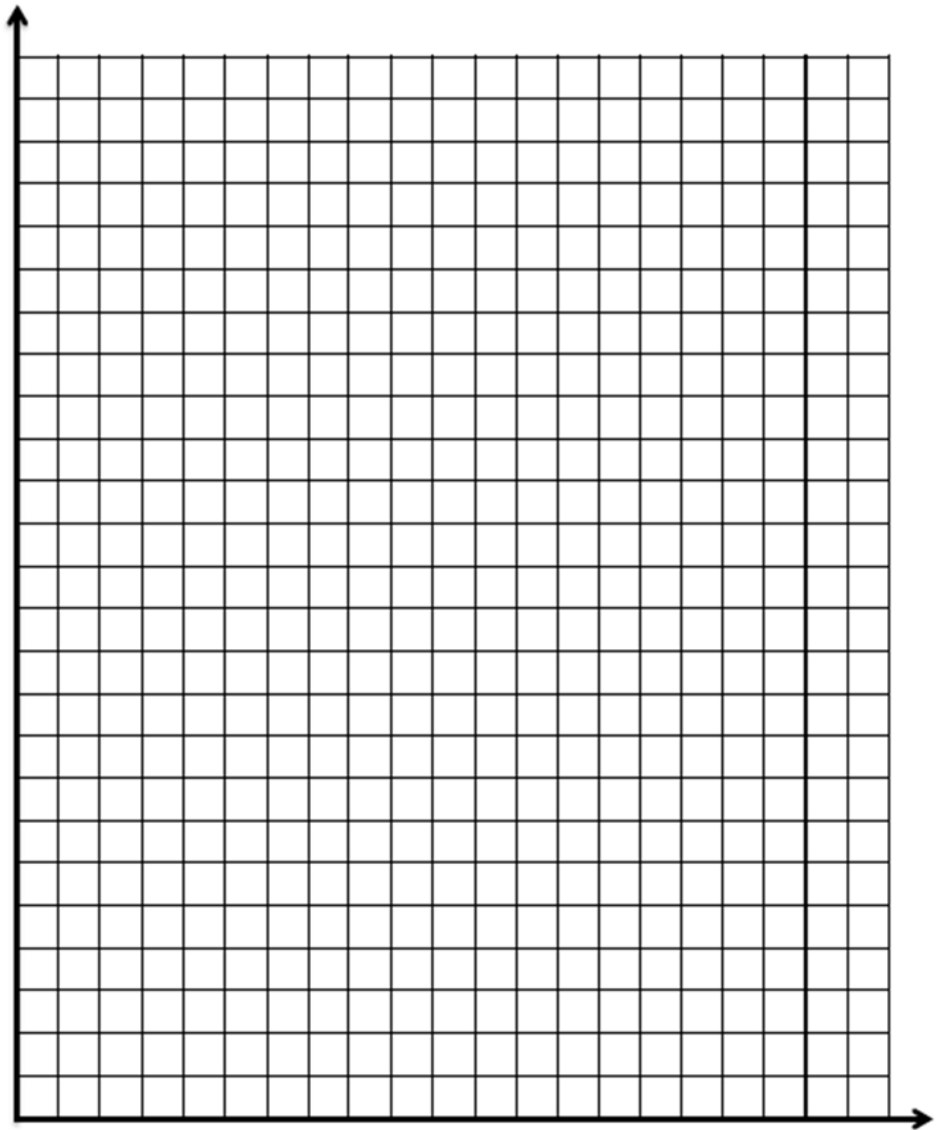
2. Create a double number line diagram to show the information in the table from #1. How many miles would be traveled in one hour? Represent the ratio of the distance traveled to the amount of time taken with an equation.



**Example 1**

Dinner service starts once the train is 250 miles away from Yonkers. What is the minimum time the players will have to wait before they can have their meal?

Hours	Miles	Ordered Pairs



## Lesson 15: A Synthesis of Representations of Equivalent Ratio Collections

### Classwork

This morning, the local television station highlighted area pets that need to be adopted. The station posted a specific website on the screen for viewers to find more information on the pets shown and the adoption process. The station producer checked the website two hours after the end of the broadcast and saw that the website had 24 views. One hour after that, the website had 36 views.

#### Exercise 1

Complete the table to show how many views the website probably had based on how many views it had two and three hours after the end of the broadcast.

Hours	1	2	3	4	5	6
Views						

#### Exercise 2

What is the constant number,  $c$ , that makes these ratios equivalent?

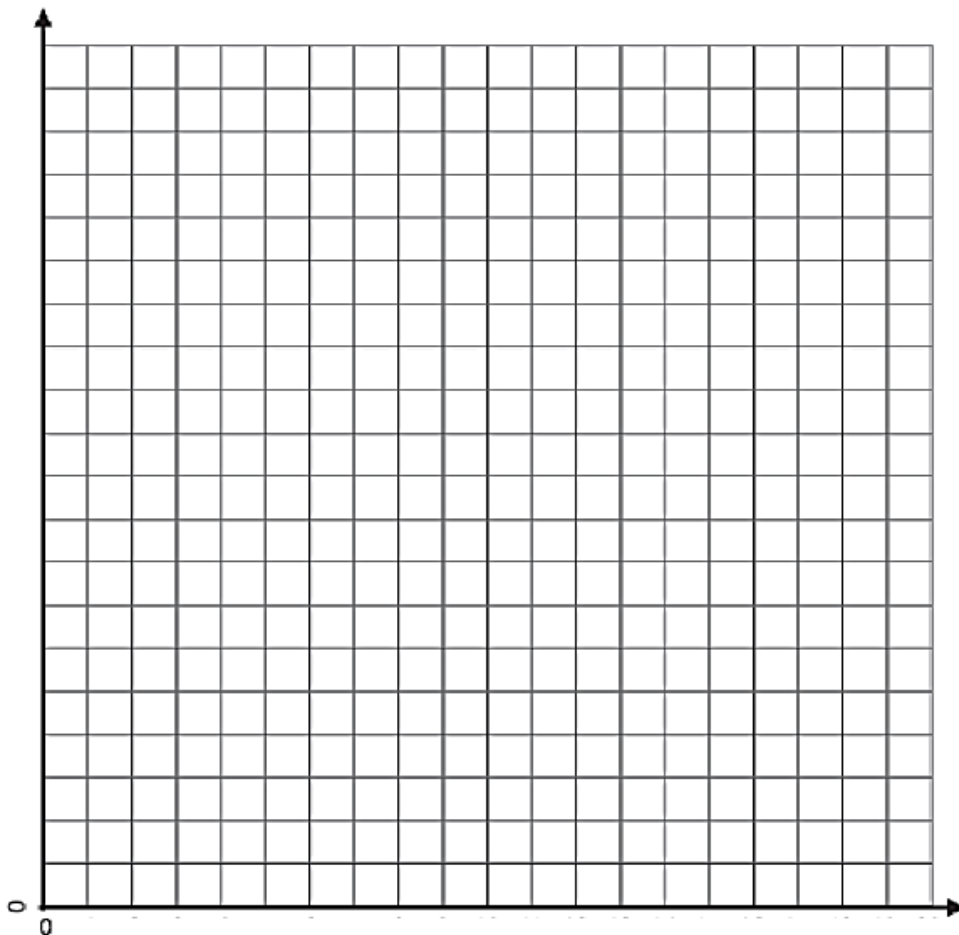
Using an equation, represent the relationship between the number of views,  $v$ , the website received and the number of hours,  $h$ , after this morning's news broadcast.

**Exercise 3**

Use the table created in Exercise 1 to identify sets of ordered pairs that can be graphed.

**Exercise 4**

Use the ordered pairs you created to depict the relationship between hours and number of views on a coordinate plane. Label your axes and create a title for the graph. Do the points you plotted lie on a line?



**Exercise 5**

Predict how many views the website will have after twelve hours. Use at least two representations (e.g., tape diagram, table, double number line diagram) to justify your answer.

**Exercise 6**

Also on the news broadcast, a chef from a local Italian restaurant demonstrated how he makes fresh pasta daily for his restaurant. The recipe for his pasta is below:

3 eggs, beaten

1 teaspoon salt

2 cups all-purpose flour

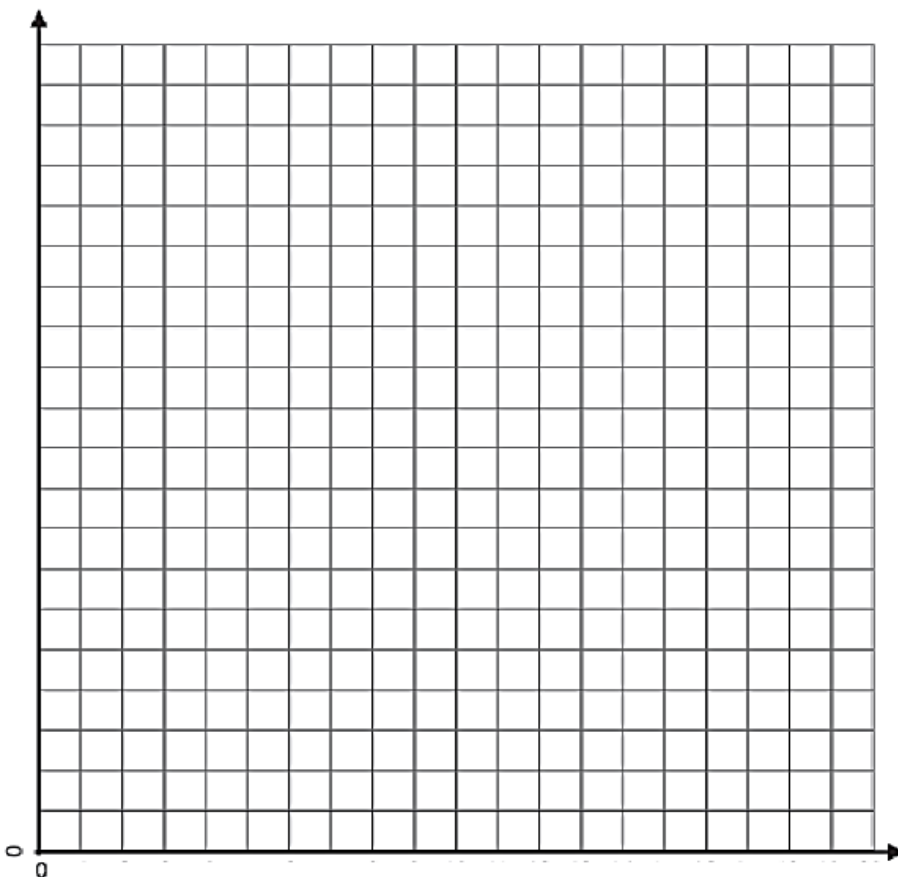
2 tablespoons water

2 tablespoons vegetable oil

Determine the ratio of the number of tablespoons of water to the number of eggs.

Complete the table below to determine ordered pairs. Use the ordered pairs to graph the relationship of the number of tablespoons of water to the number of eggs.

Tbls of Water	Number of Eggs
2	
4	
6	
8	
10	
12	



What would you have to do to the graph in order to find how many eggs would be needed if the recipe was larger and called for 16 tablespoons of water?

Demonstrate on your graph.

How many eggs would be needed if the recipe called for 16 tablespoons of water?

## Lesson 16: Ratios and Rates

### Classwork

Ratios can be transformed to rates and unit rates.

#### Example 1: Introduction to Rates and Unit Rates

Diet cola was on sale last week; it cost \$10 for every 4 packs of diet cola.

a. How much do 2 packs of diet cola cost?

b. How much does 1 pack of diet cola cost?

### Exploratory Challenge

1. Teagan went to *Gamer Realm* to buy new video games. *Gamer Realm* was having a sale: \$65 for 4 video games. He bought 3 games for himself and one game for his friend, Diego, but Teagan does not know how much Diego owes him for the one game. What is the unit price of the video games? What is the rate unit?



2. Four football fans took turns driving the distance from New York to Oklahoma to see a big game. Each driver drove at a constant speed. The group changed drivers each time they stopped for gas and recorded their driving times and distances in the table below. Use the given data to answer the following questions.

Fan	Distance (miles)	Time (hours)
Andre	208	4
Matteo	456	8
Janaye	300	6
Greyson	265	5

- i. What two quantities are being compared?
- ii. What is the ratio of the two quantities for each driver's portion of the trip? What is the associated rate?

Andre's Ratio: \_\_\_\_\_

Andre's Rate: \_\_\_\_\_

Matteo's Ratio: \_\_\_\_\_

Matteo's Rate: \_\_\_\_\_

Janaye's Ratio: \_\_\_\_\_

Janaye's Rate: \_\_\_\_\_

Greyson's Ratio: \_\_\_\_\_

Greyson's Rate: \_\_\_\_\_

- iii. For each driver in part (ii), circle the unit rate and put a box around the rate unit.

**Examples 2-3 From Ratios to Rates**

2. Dave can clean pools at a constant rate of  $\frac{3}{5}$  pools/hour.
- What is the ratio of the number of pools to the number of hours?
  - How many pools can Dave clean in 10 hours?
  - How long does it take Dave to clean 15 pools?
3. The corner store sells apples at a rate of 1.25 dollars per apple.
- What is the ratio of the amount in dollars to the number of apples?
  - Akia is only able to spend \$10 on apples. How many apples can she buy?
  - Christian has \$6 in his wallet and wants to spend it on apples. How many apples can Christian buy?



**Exercises**

Use the table below to write down your work and answers for the stations.

1.
2.
3.
4.
5.
6.

## Lesson 19: Comparison Shopping—Unit Price and Related Measurement Conversions

**Classwork:** Analyze tables, graphs, and equations in order to compare rates.

### Examples 1-2: Creating Tables from Equations

1. The ratio of cups of blue paint to cups of red paint is 1:2, which means for every cup of blue paint, there are two cups of red paint. The equation would be  $\text{red} = 2 \times \text{blue}$ , or  $r = 2b$ , where  $b$  represents the amount of blue paint and  $r$  represents the amount of red paint. Make a table of values.

Cups of Blue Paint				
Cups of Red Paint				

- Use the table to find the unit rate.
  - Where do you see the unit rate in the equation?
2. Ms. Siple is a librarian who really enjoys reading. She can read  $\frac{3}{4}$  of a book in one day. This relationship can be represented by the equation  $\text{days} = \frac{3}{4} \text{books}$ , which can be written as  $d = \frac{3}{4}b$ , where  $b$  represents the number of books and  $d$  represents the number of days. Make a table of values.

Number of Books				
Number of Days				

- Use the table to find the unit rate.
- Where do you see the unit rate in the equation?

**Example 3: Matching Tables, Graphs, and Equations**

3. Using the cards, match an equation, table, and graph that represent the same unit rate.

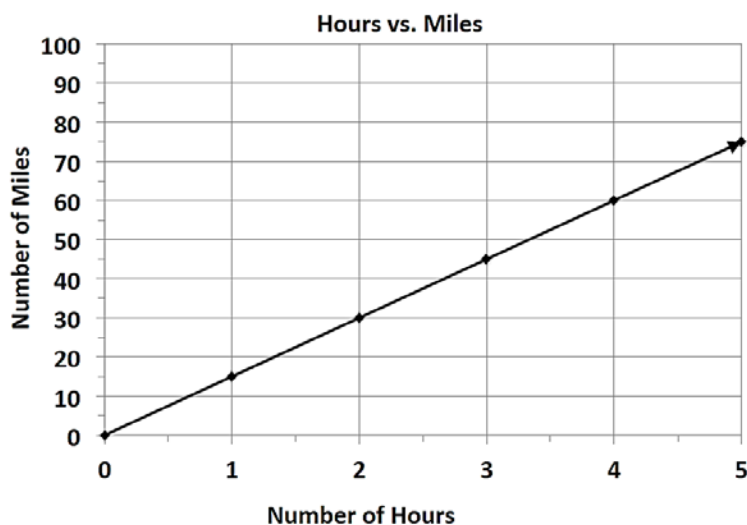
**Exercises**

1. Bryan and ShaNiece are both training for a bike race and want to compare who rides his or her bike at a faster rate. Both bikers use apps on their phones to record the time and distance of their bike rides. Bryan's app keeps track of his route on a table, and ShaNiece's app keeps track on a graph.

Bryan:

<b>Number of Hours</b>	0	3	6
<b>Number of Miles</b>	0	75	150

ShaNiece:



a. At what rate does each biker travel? Explain how you arrived at your answer.

b. If ShaNiece wants to win the race, how fast would she have to ride?

2. Braylen and Tyce both work at a department store and are paid by the hour. The manager told the boys they both earn the same amount of money per hour, but Braylen and Tyce did not agree. They each kept track of how much money they earned to determine if the manager was correct.

Braylen:

$$m = 10.50h$$

Where  $h$  represents the number of hours worked and  $m$  represents the amount of money Braylen was paid.

Tyce:

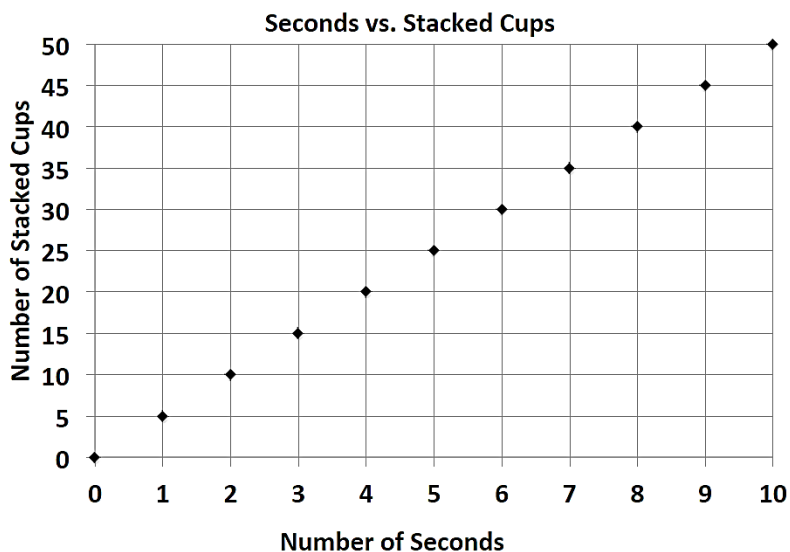
Number of Hours	0	3	6
Money in Dollars	0	34.50	69

- a. How much did each person earn in one hour?

- b. Was the manager correct? Why or why not?

3. Claire and Kate are entering a cup stacking contest. Both girls have the same strategy: stack the cups at a constant rate so that they do not slow down at the end of the race. While practicing, they keep track of their progress.

Claire:



Kate:

$$c = 4t$$

Where  $t$  represents the amount of time in seconds and  $c$  represents the number of stacked cups.

- a. At what rate does each girl stack her cups during the practice sessions?
- b. Kate notices that she is not stacking her cups fast enough. What would Kate's equation look like if she wanted to stack cups faster than Claire?



## Lesson 20: Comparison Shopping—Unit Price and Related Measurement Conversions

### Classwork

#### Exploratory Challenge

- a. Mallory is on a budget and wants to determine which cereal is a better buy. A 10-ounce box of cereal costs \$2.79, and a 13-ounce box of the same cereal costs \$3.99.
- Which box of cereal should Mallory buy?
  - What is the difference between the two unit prices?
- b. Mitch needs to purchase pop for a staff party. He is trying to figure out if it is cheaper to buy the 12-pack of soda or the 20-pack of soda. The 12-pack of soda costs \$3.99, and the 20-pack of soda costs \$5.48.
- Which pack should Mitch choose?
  - What is the difference in cost between single cans of soda from each of the two packs?

c. The table below shows the amount of calories Mike burns as he runs.

<b>Number of Miles Ran</b>	3	6	9	12
<b>Number of Calories Burned</b>	360	720		1,440

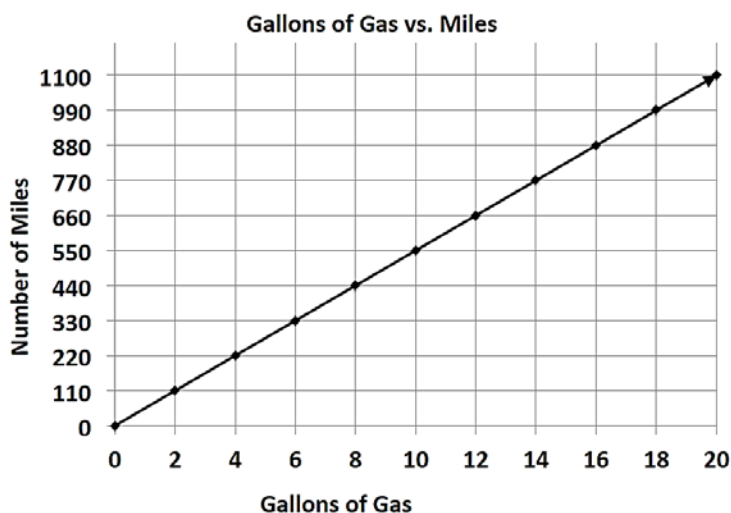
Fill in the missing part of the table.

d. Emilio wants to buy a new motorcycle. He wants to compare the gas efficiency for each motorcycle before he makes a purchase. The dealerships presented the data below.

Sports Motorcycle:

<b>Number of Gallons of Gas</b>	5	10	15	20
<b>Number of Miles</b>	287.5	575	862.5	1,150

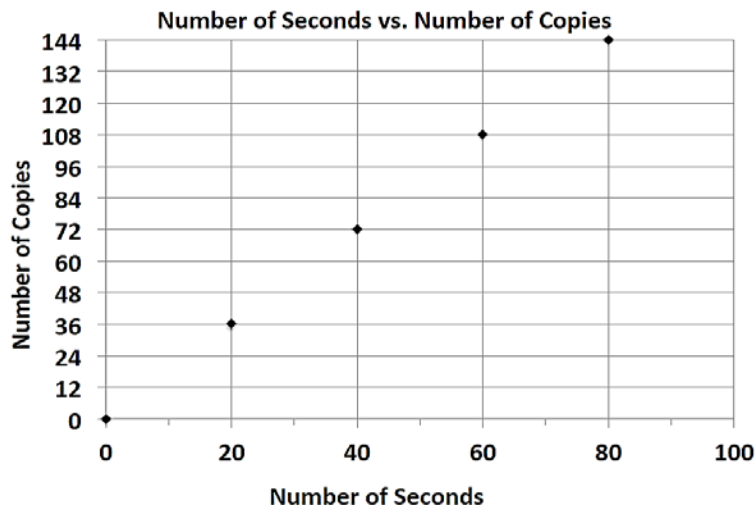
Leisure Motorcycle:



Which motorcycle is more gas efficient and by how much?

- e. Milton Middle School is planning to purchase a new copy machine. The principal has narrowed the choice to two models: SuperFast Deluxe and Quick Copies. He plans to purchase the machine that copies at the fastest rate.

SuperFast Deluxe:



Quick Copies:

$$c = 1.5t$$

Where  $t$  represents the amount of time in seconds, and  $c$  represents the number of copies

Which copier the principal should choose?

Elijah and Sean are participating in a walk-a-thon. Each student wants to calculate how much money he would make from his sponsors at different points of the walk-a-thon.

Elijah's Sponsor Plan:

Miles Walked	Money Earned
7	35
14	70
21	105
28	140

Sean's Sponsor Plan:

Miles Walked	Money Earned
6	33
12	66
18	99
24	132

Determine which student would earn more money if they both walked the same distance.

How much more money would that student earn per mile?

- f. Emilia and Miranda's mother just signed them up for a new cell phone.

Emilia:

<b>Number of Weeks</b>	3	6	9	12
<b>Number of Texts</b>	1,200	2,400	3,600	4,800

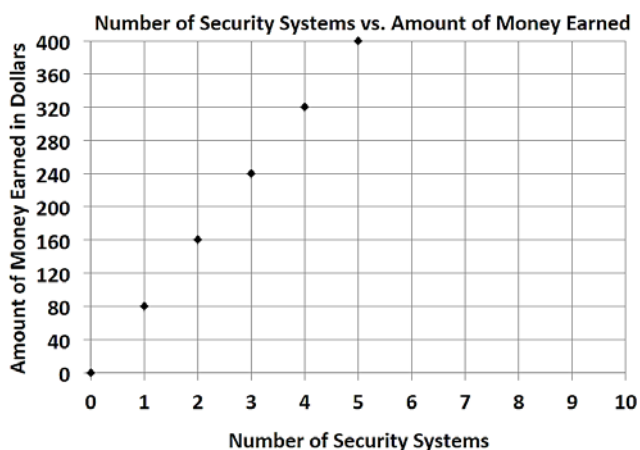
Miranda:

$m = 410w$ , where  $w$  represents the number of weeks, and  $m$  represents the number of text messages.

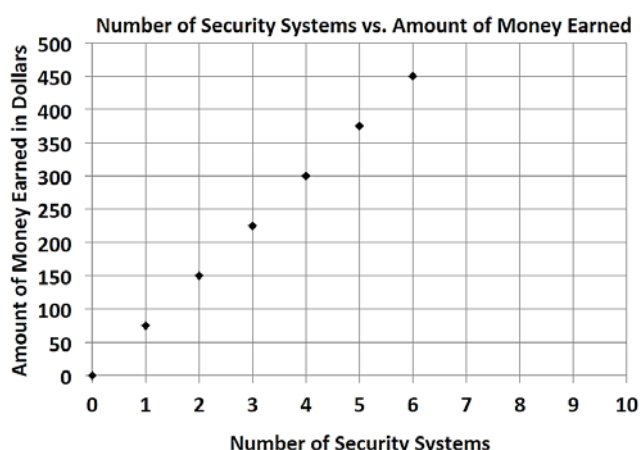
Which sister sends the most text messages? How many more text messages does this sister send per week?

- g. Zyearaye is trying to decide which security system company he will make more money working for. The graphs show the commission rate each company will pay him based on the number of security systems.

Superior Security:



Top Notch Security:



Which company pays the better commission rate? How much more money per security system would he earn by choosing the company with the better rate?

## Lesson 21: Speed, Work, and Measurement Units

### Classwork

Conversion tables contain ratios that can be used to convert units of length, weight, or capacity. You must multiply the given number by the ratio that compares the two units.

### Opening Exercise

Identify the ratios that are associated with conversions between feet, inches, and yards.

12 inches = \_\_\_\_\_ foot; the ratio of inches to feet is \_\_\_\_\_.

1 foot = \_\_\_\_\_ inches; the ratio of feet to inches is \_\_\_\_\_.

3 feet = \_\_\_\_\_ yard; the ratio of feet to yards is \_\_\_\_\_.

1 yard = \_\_\_\_\_ feet; the ratio of yards to feet is \_\_\_\_\_.

### Example 1

Work with your partner to find out how many feet are in 48 inches. Make a ratio table that compares feet and inches. Use the conversion rate of 12 inches per foot or  $\frac{1}{12}$  foot per inch.

**Example 2**

How many grams are in 6 kilograms? Use the conversion rate of 1,000 grams per kilogram.

**Exercise 1**

How many cups are in 5 quarts? Use the conversion rate of 4 cups per quart.

**Exercise 2**

How many quarts are in 10 cups?

## Lesson 22: Speed, Work, and Measurement Units

### Classwork

If an object is moving at a constant rate of speed for a certain amount of time, we can find how far the object went by multiplying the rate and the time. In mathematical language, we say, distance = rate · time.

#### Example 1

**Walker:** Substitute the walker's distance and time into the equation and solve for the rate of speed.

$$\text{distance} = \text{rate} \cdot \text{time}$$

$$d = r \cdot t$$

**Runner:** Substitute the runner's time and distance into the equation to find the rate of speed.

$$\text{distance} = \text{rate} \cdot \text{time}$$

$$d = r \cdot t$$

#### Example 2

**Part 1:** Chris Johnson ran the 40-yard dash in 4.24 seconds. What is the rate of speed? Round any answer to the nearest hundredth.

$$\text{distance} = \text{rate} \cdot \text{time}$$

$$d = r \cdot t$$

Part 2: If the runner were able to run at a constant rate, how many yards would he run in an hour? This problem can be solved by breaking it down into two steps. Make a record of your calculations.

a. How many yards would he run in one minute?

b. How many yards would he run in one hour?

It is possible to complete this same problem in one step. We can multiply the yards per second by the seconds per minute, then by the minutes per hour.

$$\text{_____} \frac{\text{yards}}{\text{second}} \cdot 60 \frac{\text{seconds}}{\text{minute}} \cdot 60 \frac{\text{minutes}}{\text{hour}} = \text{_____} \text{ yards in one hour}$$

Cross out any units that are in both the numerator and denominator in the expression because these cancel each other out.

Part 3: How many miles did the runner travel in that hour? Round your response to the nearest tenth.

Cross out any units that are in both the numerator and denominator in the expression because they cancel out.



**Exercises: Road Trip****Exercise 1**

I drove my car on cruise control at 65 miles per hour for 3 hours without stopping. How far did I go?

$$d = r \cdot t$$

$$d = \underline{\hspace{2cm}} \frac{\text{miles}}{\text{hour}} \cdot \underline{\hspace{2cm}} \text{ hours}$$

Cross out any units that are in both the numerator and denominator in the expression because they cancel out.

$$d = \underline{\hspace{2cm}} \text{ miles}$$

**Exercise 2**

On the road trip, the speed limit changed to 50 miles per hour in a construction zone. Traffic moved along at a constant rate (50 mph), and it took me 15 minutes (0.25 hours) to get through the zone. What was the distance of the construction zone?

$$d = r \cdot t$$

$$d = \underline{\hspace{2cm}} \frac{\text{miles}}{\text{hour}} \cdot \underline{\hspace{2cm}} \text{ hours}$$

**Example 3: Survival of the Fittest**

Which runs faster: a cheetah that can run 60 feet in 4 seconds or gazelle that can run 100 feet in 8 seconds?

$$\frac{\text{feet}}{\text{seconds}} = \frac{\text{feet}}{1 \text{ second}}$$

$$\frac{\text{feet}}{\text{seconds}} = \frac{\text{feet}}{1 \text{ second}}$$

**Exercise 3**

One math student, John, can solve 6 math problems in 20 minutes while another student, Juaquine, can solve the same 6 math problems at a rate of 1 problem per 4 minutes. Who works faster?

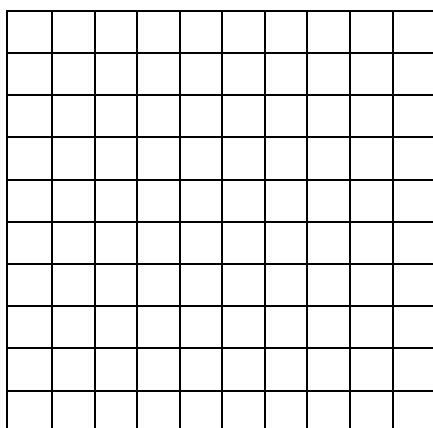
# Lesson 24: Percent and Rates per 100

## Classwork

### Exercise 1

Robb’s Fruit Farm consists of 100 acres. Three different types of apples grow there. On 25 acres, the farm grows Empire apples. McIntosh apples grow on 30%. The remainder of the farm grows Fuji apples.

Shade in the grid below to represent the portion of the farm each type of apple occupies. Use a different color for each. Create a key to identify which color represents each type of apple.



#### Color Key

Empire \_\_\_\_\_

McIntosh \_\_\_\_\_

Fuji \_\_\_\_\_

#### Part-to-Whole Ratio

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

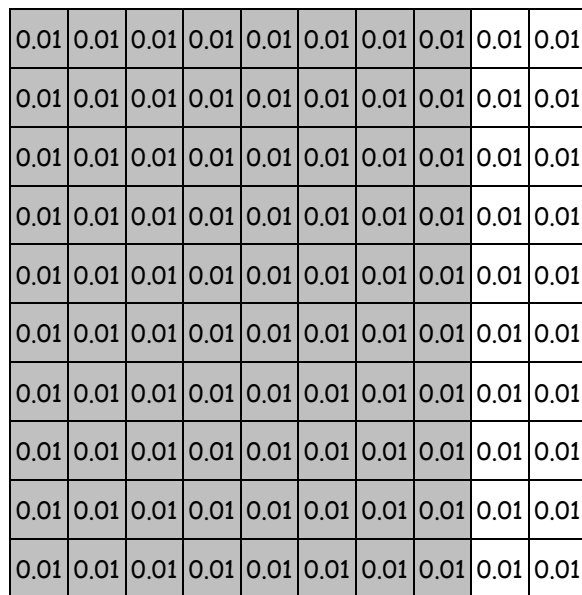
### Exercise 2

The shaded portion of the grid represents the portion of a granola bar remaining.

What percent does each block of granola bar represent?

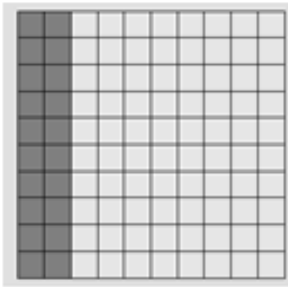
What percent of the granola bar remains?

What other ways can we represent this percent?

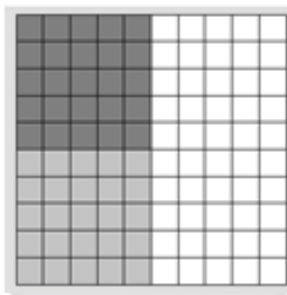


Exercise 3

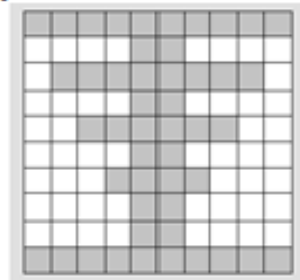
a.



b.



c.



- a. For each figure shown, represent the gray shaded region as a percent of the whole figure. Write your answer as a decimal, fraction, and percent.


Picture (a)	Picture (b)	Picture (c)

b. What ratio is being modeled in each picture?

c. How are the ratios and percentages related?

**Exercise 4**

Each relationship below compares the shaded portion (the part) to the entire figure (the whole). Complete the table.

Percentage	Decimal	Fraction	Ratio	Model
6%			6:100	
60%				
	0.55			
		$\frac{9}{10}$		
				

**Exercise 5**

Mr. Brown shares with the class that 70% of the students got an A on the English vocabulary quiz. If Mr. Brown has 100 students, create a model to show how many of the students received an A on the quiz.

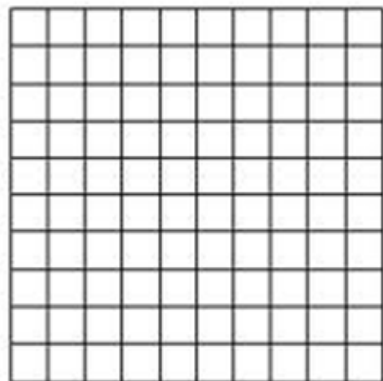
What fraction of the students received an A on the quiz?

How could we represent this amount using a decimal?

How are the decimal, the fraction, and the percent all related?

**Exercise 6**

Marty owns a lawn mowing service. He has three employees and 100 lawns to mow this week. Use the  $10 \times 10$  grid to model how the work could have been distributed between the three employees.



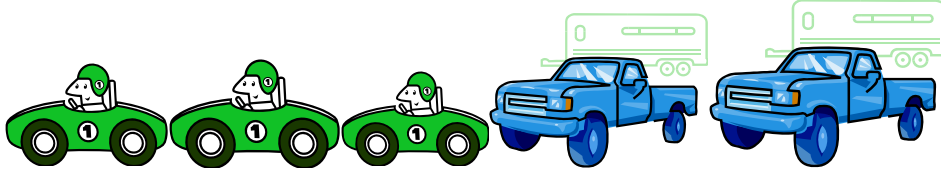
Worker	Percentage	Fraction	Decimal
Employee 1			
Employee 2			
Employee 3			

Color over the name with the same color you used in the diagram.

## Lesson 25: A Fraction as a Percent

### Classwork

#### Example 1



Sam says 50% of the vehicles are cars. Give three different reasons or models that prove or disprove Sam's statement. Models can include tape diagrams,  $10 \times 10$  grids, double number lines, etc.

How is the fraction of cars related to the percent?

Use a model to prove that the fraction and percent are equivalent.

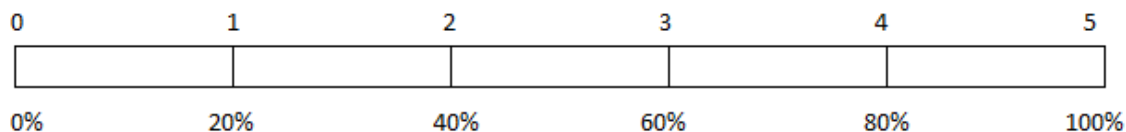
What other fractions or decimals also represent 60%?





**Exercise 2**

Use the diagram to answer the following questions.



- a) 80% is what fraction of the whole quantity?
- b)  $\frac{1}{5}$  is what percent of the whole quantity?
- c) 50% is what fraction of the whole quantity?
- d) 1 is what percent of the whole quantity?

**Exercise 3**

Maria completed  $\frac{3}{4}$  of her workday. Create a model that represents what percent of the workday Maria has worked.

What percent of her workday does she have left? How does your model prove that your answer is correct?

**Exercise 4**

Matthew completed  $\frac{3}{8}$  of his workday. Create a model that represents the percent of the workday he has finished.

How can you use the percent to get the decimal of the workday Matthew has completed?

**Exercise 5**

Complete the conversions from fraction to decimal to percent.

Fraction	Decimal	Percent
$\frac{1}{8}$		
	0.35	
		84.5%
	0.325	
$\frac{2}{25}$		

## Lesson 26: Percent of a Quantity

### Classwork

#### Example 1

Five of the 25 girls on Alden Middle School's soccer team are seventh-grade students. Find the percentage of seventh graders on the team. Show two different ways of solving for the answer. One of the methods must include a diagram or picture model.

#### Example 2

The Alden Middle School girls' soccer team won 80% of its games this season. If the team won 12 games, how many games did it play? Solve the problem using at least two different methods.

**Exercises**

1. There are 60 animal exhibits at the local zoo. What percent of the zoo's exhibits does each animal class represent?

Exhibits by Animal Class	Number of Exhibits	Percent of the Total Number of Exhibits
Mammals	30	
Reptiles & Amphibians	15	
Fish & Insects	12	
Birds	3	

2. A sweater is regularly \$32. It is 25% off the original price this week.
- Would the amount the shopper saved be considered the part, whole, or percent?
  
  
  
  
  
  
  
  
  
  
  - How much would a shopper save by buying the sweater this week? Show two methods for finding your answer.



## Lesson 27: Solving Percent Problems

### Classwork

#### Example 1

Solve the following three problems.

Write the words PERCENT, WHOLE, or PART under each problem to show which piece you were solving for.

$$60\% \text{ of } 300 = \underline{\hspace{2cm}} \quad 60\% \text{ of } \underline{\hspace{2cm}} = 300 \quad 60 \text{ out of } 300 = \underline{\hspace{2cm}}\%$$

\_\_\_\_\_

How did your solving method differ with each problem?

**Exercise**

Use models, such as  $10 \times 10$  grids, ratio tables, tape diagrams, or double number line diagrams, to solve the following situation.

Priya is doing her back-to-school shopping. Calculate all of the missing values in the table below, rounding to the nearest penny, and calculate the total amount Priya will spend on her outfit after she receives the indicated discounts.

	<b>Shirt</b> (25% discount)	<b>Pants</b> (30% discount)	<b>Shoes</b> (15% discount)	<b>Necklace</b> (10% discount)	<b>Sweater</b> (20% discount)
<b>Original Price</b>	\$44			\$20	
<b>Amount of Discount</b>		\$15	\$9		\$7

What is the total cost of Priya's outfit?



## Lesson 28: Solving Percent Problems

### Classwork

#### Example

If an item is discounted 20%, the sale price is what percent of the original price?

If the original price of the item is \$400, what is the dollar amount of the discount? How much is the sale price?

#### Exercise

The following items were bought on sale. Complete the missing information in the table. Use models to help you.

Item	Original Price	Sale Price	Amount of Discount	Percent Saved	Percent Paid
Television		\$800		20%	
Sneakers	\$80			25%	
Video Games		\$54			90%
Snack Bar		\$1.70	\$0.30		

